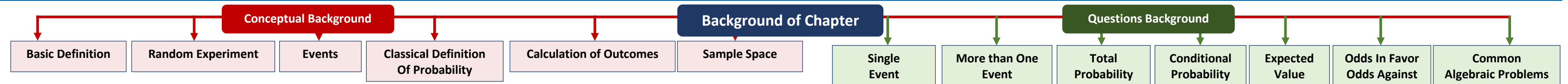


Chapter-12

PROBABILITY



Conceptual Background

1. Basic Definition

Probability means **Chance** of "Occurrence" of any Particular Event

2. Random Experiment

Experiment Whose Outcome is **Not Known** is Known as **Random Experiment**
E.g.: Tossing a Coin, Rolling a Dice

3. Events

Experiment Whose Outcome is **Not Known** is Known as **Random Experiment**
E.g.: Tossing a Coin, Rolling a Dice

Sure Event

Event Whose Chance of Happening is 100%

$P(A) = 1$

Impossible Event

Event Whose Chance of Happening is 0 %

$P(A) = 0$

Equiprobable Event

When Probability of Events are Same then Events are Equiprobable Event

$P(A) = P(B)$

Independent Event

When **Happening** or **Non-Happening** of **One Event Does Not Effect** Happening or Nor Happening of **another Events** then events are independent.

Dependent Event

When **Happening** or **Non-Happening** of **One Event Effect** Happening or Nor Happening of **another Events** then events are dependent.

Mutually Exclusive

Events are said to M.E when then have **Nothing Common**

Exhaustive

Events are Exhaustive if Addition of all Give us **Sample Space**
 $A_1 + A_2 + A_3 = S.S$

4. Classical Definition of Probability

$P[A] = \frac{m}{n} = \frac{\text{no of elements in favour of A}}{\text{Total no of Outcomes}}$

1. $0 \leq m \leq n$

2. $\frac{0}{n} \leq \frac{m}{n} \leq \frac{n}{n}$

3. $0 \leq P[A] \leq 1$

4. $P[A] + P[\bar{A}] = 1$

Conclusion Note

Students: Sir can we conclude from above that probability calculation is based on **NO OF OUTCOMES**

Aman Sir: Yes Of Course, we need to two things
No of favourable Outcomes & Total no of Outcomes

Sample Space Technique

Use of Combination Technique

Aman Sir Observation

We have Observed that Students Usually Get **Confused** that when to Draw **Sample Space** & When to Use **Combination** So Now Let's Remove this Confusion

Sample Space Technique

When Data is **Small** & it **Practically Possible to Count** the Outcomes then Only, we will Prefer Sample Space Technique
Example:

Example: From the letters of the word "MOBILE", a letter is Selected at Random. The Probability it is Vowel is?
Solu:

Use of Combination Technique

When Data is **Small** & it **Practically Possible to Count** the Outcomes then Only, we will Prefer Sample Space Technique
Example:

Example: If **2 Alphabets** are selected at Random from a word "MOBILE", What is the probability that alphabets are Vowels?
Solu:

Sample Space Technique

(MO, MB, MI, ML, ME, OB, **OI**, OL, **OE**, BI, BL, BE, IL, **IE**, LE)

Combination Technique

5. Sample Space

We will Create Sample Space in question if it small and possible ...So let's **Learn** Some **Basic Sample Space**

Coins

Coins Sample Space: No of Outcomes = (Outcomes of Single Coin)^{No of Coins}

1 Coin {H, T}

2 Coin {HH, HT, TH, TT}

3 Coin {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Dice

Dice Sample Space: No of Outcomes = (Outcomes of Single Dice)^{No of Dices}

1 Dice $S = \{1, 2, 3, 4, 5, 6\}$

2 Dice (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

Shortcut

$P(\text{Sum}) = \frac{x-1}{36} \quad 2 \leq x \leq 7$

$P(\text{Sum}) = \frac{13-x}{36} \quad 8 \leq x \leq 12$

Example: If two dice is rolled find the probability of getting
(a) Sum as 2
(b) Sum as 9

Cards

Cards Sample Space 52 Cards (Joker is not a Card)

BLACK CARDS [26 Cards]

SPADE (13)	CLUB (13)
A	A
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
J	J
Q	Q
K	K

Denomination Card

Face Card

RED CARDS [26 Cards]

HEART (13)	DIAMOND (13)
A	A
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
J	J
Q	Q
K	K

Face Cards = (J+Q+K) = 3×4 = 12

Honor Cards = Ace + FC = 4×4 = 16

Phase-1

Question Background

6. Single Event

We have 2 Alternatives to Solve Question

C.D.P

When to think?

When we Can Count No of Outcome Either by-

- Sample Space or
- Use of Combination.

Example:

A dice is rolled. What is probability that a number 1 or 6 may appear on the face?

Solu:

G.A.T

When to think?

Apply When –

- We Can't Count Fav & Total Outcomes because Direct Probabilities are Given
- Que Demands At least One or Only One Task
- Use of Word "OR" (it mean's At least Once)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

A

We Can Count Intersecting Outcome

B

$P(A \cap B)$ is Directly Given

C

$P(A \cap B)$ is Neither we Can Compute nor Given but Task are INDEPENDENT

After GMT

Clarification Note

If Events are Mutually Exclusive then Intersection is Zero $\{P(A \cap B = 0)\}$

$P(A \cup B) = P(A) + P(B)$

In Case of 3 Events

$P(A \cup B) = P(A) + P(B) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Example:

A drawer contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one item is chosen at random, what is the probability that it is rusted or a bolt?

Solu:

A

Example:

A drawer contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one item is chosen at random, what is the probability that it is rusted or a bolt?

Solu:

B

Example:

Two cards are drawn from a pack of 52 cards. What is the probability that either both are red or both are kings?

Solu:

C

Example:

A bag contains 12 balls which are numbered from 1 to 12. If a ball is selected at random, what is the probability that the number of the ball will be a multiple of 5 or 6?

Solu:

Example:

The Probability that a football team winning a match at Kolkata is $\frac{3}{5}$ and winning a match at Bengaluru is $\frac{6}{7}$; the probability of the team winning at least one match is _____

Solu:

After GMT

7. Multiple Event

In Succession

Two Consecutive Draw

One by One

And Another Draw

Replacement/Without Rep.

G.M.T

Independent Event

(When there is No Change in Sample Space of 2nd Event)

Example:

A bag contains 10 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that first is white and second is black?

Solu:

Example:

A bag contains 5 white, 7 red and 8 black balls. If four balls are drawn one by one without replacement, find the probability of getting all white balls.

Solu:

Example:

Find the probability of drawing a diamond card in each of the two consecutive draws from a well shuffled pack of cards, if the card drawn is not replaced after the first draw.

Solu:

Example

A box contains 5 white and 7 black balls. Two successive drawn of 3 balls are made

- (i) With replacement
- (ii) without replacement.

The probability that the first draw would produce white balls and the second draw would produce black balls are respectively

Solu:

NOTE

Till Now we have Solved Question where Order of Draw was Given in Question What if Order of Draw is Not Given

Example:

A bag contains 5 blue and 4 red balls. If two balls are drawn random one by one without replacement, find the probability of getting one ball is blue and one ball is red?

Sol:

Dependent Event

(When there is Change (Reduction/Addition) in Sample Space of 2nd Event)

8. Total Probability

E-1

E-2

Required Probability (Blue Ball)

When there are More than One Event Which Satisfy the Demands of the Question's Required Probability then when we Will Calculate the Probability of Each Event and Add then to Get Total Probability

Example:

A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. One bag is selected at random. from the selected bag, one ball is drawn. Find the probability that the ball drawn is red.

Solu:

Example:

A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn from first bag without noticing its colour is put in second bag. A ball is then drawn from the second bag. Find the probability that the ball drawn is blue

Solu:

Important Note

Questions Demands At least One (Events are Independent)

We Start thinking about G.A.T & G.M.T

Shortcut

Example:

A problem in probability was given to three CA students A, B and C whose chances of solving it are 1/3, 1/5 and 1/2 respectively. What is the probability that the problem would be solved?

Solu:

CA Foundation

Quantitative Aptitude

Phase-2

Question Background

9. Conditional Probability

When to think?

Chance of Occurrence of **ONE EVENT** given that **Another event** has **Already Occurred**



Example: In a class 40% students read Mathematics, 25 % Biology and 15 % both Mathematics and Biology. One student is select at random. The probability that he reads Mathematics if it is known that he reads Biology is

Solution:

Example: The Probability of a student passing in science is $\frac{4}{5}$ and the probability of the student passing in both science & maths is $\frac{1}{2}$. What is the probability of that student passing in maths knowing that he passed in science?

Solution:

10. Expected Value

Expected Value $E(x)$ or Mathematical Expectation of a random variable is the sum of products of the different values taken by the random variable and the corresponding probabilities.

Variance

Important Highlights

1. Expectation of constant $K = K$
2. $E[x + y] = E[x] + E[y]$
3. $E[kx] = kE[x]$
4. $E[x.y] = E[x]. E[y]$

Example: If a random variable x assumes the values 0, 1 and 2 with probabilities 0.30, 0.50 and 0.20, then its expected value is

Solution:

Example: The probability that there is at least one error in an account statement prepared by 3 persons A, B and C are 0.2, 0.3 and 0.1 respectively. If A, B and C prepare 60, 70 and 90 such statements, then the expected number of correct statements

Solution:

Example: The probability distribution of a random variable x is given below:

x:	1	2	4	5	6
p:	0.15	0.25	0.20	0.30	0.10

What is the standard deviation of x?

Solution:

11. Odds in Favor /Against

Ratio is Given
Check weather Given Ratio is of

Odds in Favor

Odds Against

Example: The odds in Favor of one [^{1st}] student passing a test are 3: 7, the odds against another student passing are 3:5, the probability that

- (a) Both Pass
(b) Only the first Student Pass
(c) At Least one of them will pass

Solution:

Summary Notes



Question Bank

Single Event

- Two balls are drawn from a bag containing 5 white and 7 black balls at random. What is the probability that they would be of different colors?
(a) 35/66 (b) 30/66
(c) 12/66 (d) None of these
- X and Y stand in a line with 6 other people. What is the probability that there are 3 persons between them?
(a) 1/5 (b) 1/6
(c) 1/7 (d) 1/3
- A number is selected at random from the first 1000 natural numbers. What is the probability that the number so selected would be a multiple of 7 or 11?
(a) 0.25 (b) 0.32
(c) 0.22 (d) 0.33
- Probability that a truck stopped at a roadblock will have faulty brakes or badly worn tires are 0.23 and 0.24 respectively. Also, the probability is .38 that a truck stopped at a roadblock will have faulty brakes and or badly working tires. What is the probability that a truck stopped at the roadblock will have faulty brakes as well as badly worn tires?
(a) 0.45 (b) 0.09
(c) 0.62 (d) None of These
- Two cards are drawn from a pack of 52 cards. What is the probability that either both are red or both are kings?
(a) 55/120 (b) 55/221
(c) 45/78 (d) None of These
- The probability that a person will get an electric contract is $\frac{2}{5}$ and the probability that he will not get plumbing contract is $\frac{4}{7}$. If the probability of getting at least one contract is $\frac{2}{3}$ what is the probability that he will get both?
(a) 19/105 (b) 17/105
(c) 21/105 (d) None of These
- The probability that a person visiting a dentist will have his teeth cleaned is 0.44, the probability that he will have a cavity filled is 0.24. The probability that he will have his teeth cleaned or a cavity filled is 0.60. What is the probability that a person visiting a dentist will have his teeth cleaned and cavity filled?
(a) 0.06 (b) 0.08
(c) 0.8 (d) None of These
- Four digits 1, 2, 4 and 6 are selected at random to form a four-digit number. What is the probability that the number so formed, would be divisible by 4?
(a) 1/2 (b) 1/5
(c) 1/4 (d) 1/3
- Find the probability that a leap year, selected at random, will contain 53 Sundays.
(a) 1/2 (b) 1/5
(c) 1/4 (d) 2/7
- A card is drawn from a deck of 52 cards. Find the probability of getting a king or a heart or a red card.
(a) 5/13 (b) 7/13
(c) 6/15 (d) None of These

Multiple Event

- A bag contains 8 red and 5 white balls. Two successive draws of 3 balls are made without replacement. The probability that the first draw will produce 3 white balls and the second 3 red balls is
(a) 5/223 (b) 6/257
(c) 7/429 (d) 3/548
- A police-man fires four bullets on a dacoit. The probability that the dacoit will be killed by one bullet is 0.6. What is the probability that the dacoit is still alive?
(a) 0.0256 (b) 0.8954
(c) 0.5623 (d) None of These
- A bag contains 10 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that first is white and second is black?
(a) 5/7 (b) 1/4
(c) 7/9 (d) 3/5
- Find the probability of drawing a diamond card in each of the two consecutive draws from a well shuffled pack of cards, if the card drawn is not replaced after the first draw.
(a) 5/17 (b) 1/16
(c) 1/17 (d) 3/17
- A bag contains 5 white, 7 red and 8 black balls. If four balls are drawn one by one without replacement, find the probability of getting all white balls.
(a) 5/969 (b) 6/969
(c) 7/969 (d) 1/969
- A bag contains 19 tickets, numbered from 1 to 19. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show even numbers.
(a) 5/17 (b) 1/16
(c) 4/19 (d) 3/17

GAT + Independent Event (G.M.T)

- A problem in mathematics is given to 3 students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that the problem is solved?
(a) 3/5 (b) 3/4
(c) 5/6 (d) None of These
- A can solve 90% of the problems given in a book and B can solve 70%. What is the probability that at least one of them will solve the problem, selected at random from the book?
(a) 0.97 (b) 0.89
(c) 0.56 (d) None of These
- A machine operates if all of its three components function. The probability that the first component fails during the year is 0.14, the second component fails is 0.10 and the third component fails is 0.05. What is the probability that the machine will fail during the year?
(a) 0.2647 (b) 0.8954
(c) 0.5623 (d) None of These
- A bag contains 5 white, 7 red and 8 black balls. Four balls are drawn one by one with replacement, what is the probability that at least one is white?
(a) $1 - (\frac{3}{4})^5$ (b) $1 - (\frac{3}{4})^4$
(c) $1 - (\frac{5}{4})^4$ (d) none of These

Total Probability

- There are two urns. The first urn contains 3 red and 5 white balls whereas the second urn contains 4 red and 6 white balls. A ball is taken at random from the first urn and is transferred to the second urn. Now another ball is selected at random from the second arm. The probability that the second ball would be red is
(a) 7/20 (b) 35/88
(c) 17/52 (d) 3/20
- There are three boxes with the following composition:
Box I: 5 Red + 7 White + 6 Blue balls
Box II: 4 Red + 8 White + 6 Blue balls
Box III: 3 Red + 4 White + 2 Blue balls
If one ball is drawn at random, then what is the probability that they would be of same colour?
(a) 89/729 (b) 97/729
(c) 82/729 (d) 23/32
- There are two boxes containing 5 white and 6 blue balls and 3 white and 7 blue balls respectively. If one of the boxes is selected at random and a ball is drawn from it, then the probability that the ball is blue is
(a) 115/227 (b) 83/250
(c) 137/220 (d) 127/250
- A packet of 10 electronic components is known to include 2 defectives. If a sample of 4 components is selected at random from the packet, what is the probability that the sample does not contain more than 1 defective?
(a) 1/3 (b) 2/3
(c) 13/15 (d) 3/15
- Tom speaks truth in 30 percent cases and Harry speaks truth in 25 percent cases. What is the probability that they would contradict each other?
(a) 0.325 (b) 0.400
(c) 0.925 (d) 0.075
- There are three persons aged 60, 65 and 70 years old. The survival probabilities for these three persons for another 5 years are 0.7, 0.4 and 0.2 respectively. What is the probability that at least two of them would survive another five years?
(a) 0.425 (b) 0.456
(c) 0.392 (d) 0.388

Odd in Favor/Against

- The odds in Favor of one student passing a test are 3:7. The odds against another student passing at are 3:5. The probability that both pass is
(a) $\frac{7}{16}$ (b) $\frac{21}{80}$
(c) $\frac{9}{80}$ (d) $\frac{3}{16}$
- The odds in Favor of one student passing a test are 3:7. The odds against another student passing at are 3:5. The probability that both fail is
(a) $\frac{7}{16}$ (b) $\frac{21}{80}$
(c) $\frac{9}{80}$ (d) $\frac{3}{16}$

Conditional Probability

- For a group of students, 30 %, 40% and 50% failed in Physics, Chemistry and at least one of the two subjects respectively. If an examinee is selected at random, what is the probability that he passed in Physics if it is known that he failed in Chemistry?
(a) 1/2 (b) 1/3
(c) 1/4 (d) 1/6
- Ten cards numbered 1 through 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?
(a) 3/10 (b) 6/13
(c) 4/7 (d) 1/6
- Assume that each child born is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?
(a) 1/2 and 1/3 (b) 2/5 and 1/3
(c) 1/2 and 2/3 (d) None of These
- In a class 40 % students read Mathematics, 25 % Biology and 15 % both Mathematics and Biology. One student is select at random. The probability that he reads Mathematics if it is known that he reads Biology is
(a) 2/5 (b) 3/5
(c) 4/5 (d) none
- The probability that there is at least one error in an account statement prepared by 3 persons A, B and C are 0.2, 0.3 and 0.1 respectively. If A, B and C prepare 60, 70 and 90 such statements, then the expected number of correct statements
(a) 170 (b) 176
(c) 178 (d) 180

Expected Value

- A bag contains 6 white and 4 red balls. If a person draws 2 balls and receives Rs. 10 and Rs. 20 for a white and red balls respectively, then his expected amount is
(a) Rs.25 (b) Rs. 26
(c) Rs.29 (d) Rs. 28
- The probability of winning of a person is 6/11 and at a result he gets Rs. 77/-. The expectation of this person is
(a) Rs. 35/- (b) Rs. 42/-
(c) Rs.58/- (d) none
- If $2x + 3y + 4 = 0$ and $v(x) = 6$ then $v(y)$ is
(a) 8/3 (b) 9
(c) -9 (d) 6
- The probability distribution of a random variable is as follows:

X:	1	2	4	6	8
P:	k	2k	3k	3k	k

The variance of x is
(a) 2.1 (b) 4.41
(c) 2.32 (d) 2.47

Algebraic Problems

- Given that for two events A and B, $P(A) = 3/5$, $P(B) = 2/3$ and $P(A \cup B) = 3/4$, what is $P(\frac{A}{B})$?
(a) 0.655 (b) 13/60
(c) 31/60 (d) 0.775
- Given that $P(a) = 1/2$, $P(B) = 1/3$, $P(A \cap B) = 1/4$, what is $P(A/B)$?
(a) 1/2 (b) 7/8
(c) 5/8 (d) 2/3
- If $P(A) = p$ and $P(B) = q$, then
(a) $P(A/B) \leq p/q$ (b) $P(A/B) \geq p/q$
(c) $P(A/B) \leq q/p$ (d) None of these
- If $P(A) = 2/3$, $P(B) = 3/4$, $P(A/B) = 2/3$, then what is $P(B/A)$?
(a) 1/3 (b) 2/3
(c) 3/4 (d) 1/2

Answer Sheet

1.	a	2.	c	3.	c
4.	b	5.	b	6.	b
7.	b	8.	d	9.	d
10.	b	11.	c	12.	a
13.	b	14.	c	15.	d
16.	c	17.	b	18.	a
19.	a	20.	b	21.	b
22.	a	23.	c	24.	c
25.	b	26.	d	27.	d
28.	b	29.	a	30.	c
31.	a	32.	b	33.	c
34.	d	35.	b	36.	a
37.	b	38.	d	39.	c
40.	b	41.	c		